

① Poisson Distribution

\sum If we will consider the summation of Poisson distribution $P(x) = \frac{e^{-m} m^x}{x!}$ where $x=0, 1, 2, 3, \dots, \infty$

Proof: $\rightarrow \sum P(x) = P(0) + P(1) + P(2) + P(3) + \dots \infty$
 $= \frac{e^{-m} m^0}{0!} + \frac{e^{-m} m^1}{1!} + \frac{e^{-m} m^2}{2!} + \frac{e^{-m} m^3}{3!} + \dots \infty$

$= e^{-m} \left(1 + \frac{m}{1} + \frac{m^2}{2!} + \frac{m^3}{3!} + \dots \right)$ we know that $e^x = 1 + \frac{x}{1} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$
 $= e^{-m} \cdot e^m$
 $= e^0$
 $= 1$

Q \rightarrow In a certain factory turning out optical lenses, there is a small chance $\frac{1}{500}$ for any one lens to be defective. The lenses are supplied in packing of 10. Use Poisson distribution to calculate the approximate number of packets containing no defective, one defective, two defective, three defective lenses respectively in a consignment of 20,000 packets. Given that $e^{-0.02} = 0.9802$.

Solution: \rightarrow we know from question:
 Total Number of packets = 20,000
 1 packet contains 10 lens $n=10$
 $p =$ probability of a defective optical lens = $\frac{1}{500}$

Mean

$$\begin{aligned} \text{mean} &= np \\ &= 18 \times \frac{1}{500} \\ &= 0.02 \end{aligned}$$

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$$\begin{aligned} P(X=r) &= \frac{e^{-m} m^r}{r!} \\ &= \frac{e^{-0.02} (0.02)^r}{r!} \end{aligned}$$

where r denotes the number of defective pens.

Hence, in a consignment of 20,000 packets, the frequency of packets containing r defective pens

$$= 20000 \times \frac{e^{-0.02} (0.02)^r}{r!}$$

No of packets containing no defective pens

$$\begin{aligned} &= \frac{20000 \times e^{-0.02} (0.02)^0}{1!} \\ &= 20,000 \times 0.9802 \\ &= 19604 \end{aligned}$$

No of packets containing 1 defective pens

$$\begin{aligned} &= \frac{20,000 \times e^{-0.02} (0.02)^1}{1!} \\ &= 19604 \times 0.02 \\ &= 392.08 \\ &= 392 \end{aligned}$$

No of packets containing 2 defective pens

$$\begin{aligned} &= \frac{20,000 \times e^{-0.02} (0.02)^2}{2!} \\ &= 3.9208 \end{aligned}$$

No of packets containing 3 defective pens

$$= \frac{20000 \times e^{-0.02} (0.02)^3}{3!} = 0$$